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Dynamic thermal stresses in functionally graded thick hollow cylinders by a graded finite element method and 2D axisymmetric elasticity

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ABSTRACT

In this research, an axisymmetric thick hollow cylinder made of functionally graded materials under internal thermal shock based on classical theory of linear thermoelasticity is considered. The cylinder is made of a combined ceramic-metal material and its material is graded through the thickness direction according to a power law distribution. The governing equations are based on 2D-axisymmetric theory of elasticity and graded finite element method based on Rayleigh- Ritz energy formulation is used to model the problem. To obtain transient temperatures, Crank- Nicolson algorithm is used and then Newmark direct integration method is used to obtain time history of displacements and stresses. Distribution of displacements and stresses for different power law exponents is investigated.

1. 1. INTRODUCTION

Functionally graded materials (FGMs) are advanced composite materials in which the material properties vary continuously from one surface to the other. The concept of FGMs was first presented by a group of materials scientists in Japan [1]. FGMs are usually made of a combined ceramic-metal material to achieve the desired properties. The continuous variation of the material properties eliminates interface problems, minimizes thermal stress concentrations, and causes a more smooth stress distribution. FGMs have been used in numerous applications because of their high mechanical strength and high thermal resistant. Thick hollow cylinders appear in many fields of engineering, such as civil, mechanical, and aerospace engineering. Therefore, it is important to study the dynamic thermal stresses caused by thermal shocks in thick hollow cylinders made of FGMs.

To date, a wide range of studies has been carried out on thermal stress analysis of FGM cylinders and shells [2- 35]. For example, Guo and Noda [3] using an analytical method investigated the thermal stresses of a thin FGM cylindrical shell subjected to a thermal shock. They used a perturbation method to solve the

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thermal diffusion equation. Ying and Wang [7] obtained an exact solution for two-dimensional elastodynamic analysis of finite hollow cylinder excited by non-uniform thermal shock. They employed the expansion of trigonometric series method and the separation of variables technique. Hosseini [8] studied the coupled thermo-elasticity behavior of FG thick hollow cylinders with finite length under thermal shock load. The coupled thermo-elastic equations have been considered based on Green–Naghdi theory. The cylinder was assumed to be made of many isotropic sub-cylinders across the thickness. The Galerkin Finite Element and Newmark Methods have been used to analyze the cylinder. Bahtui and Eslami [9] using a second-order shear deformation shell theory and Galerkin finite element formulation studied the coupled thermoelasticity problem of a FG cylindrical shell under impulsive thermal shock load. Shariyat et al. [11-13] studied nonlinear thermoelasticity, vibration, and stress wave propagation analyses of plane strain thick-walled cylinders made of FGMs with temperature-dependent properties. Peng and Li [15, 16] studied the thermo-elastic problem of FGM hollow cylinders with material properties of arbitrary nonhomogeneity by solving a Fredholm integral equation. Shao and Wang [18] investigated three-dimensional thermo-elastic analysis of a FG cylindrical panel with finite length under non-uniform mechanical and steady-state thermal loads. They obtained analytical solutions for the temperature and stress fields in terms of trigonometric and power series for the simply supported boundary conditions. Asgari and Akhlaghi [19, 20] using graded finite element method studied the transient thermal stresses in a thick hollow cylinder with finite length made of two-dimensional functionally graded material based on classical theory of thermo-elasticity. Safari et al. [27] presented an analytical method to study the dynamic behavior of thermo-elastic stresses in a finite-length FG thick hollow cylinder under thermal shock loading. They used Laplace transform and series solution to solve the thermo-elastic Navier equations in displacement form. Hesseini et al. [28] studied heat wave propagation and coupled thermo-elasticity without energy dissipation in FG plane strain thick hollow cylinder based on Green–Naghdi theory. They used the Galerkin finite element method and Newmark finite difference method to solve the problem. Najibi et al. [29] investigated nonlinear transient thermo-elastic analysis for a thick hollow 1D-FGM axisymmetric cylinder with finite length using higher-order graded finite element method.

As described above, in the most of cases the studies are made about the steady state and transient thermal stresses of FGM cylindrical shells and thick cylinders. To our knowledge, little attention is given to the analysis of thermo-elastic behavior of FGM thick hollow cylinders under thermal shocks. In other words, dynamic thermal stresses or thermally induced vibration of these structures is not investigated, before. Therefore, in this paper, classical theory of thermo-elasticity and graded finite element method (GFEM) based on Rayleigh–Ritz energy formulation have been used to study the dynamic thermal stresses in thick hollow cylinders made of FGMs. In GFEM, temperature, displacements and material properties interpolated using the same shape functions and this gives continuous and smooth variation of stress field than using conventional FEM. Using this method, the effects of different power law exponent on distribution of displacements and stresses have been considered.

2. Governing equations

2.1. Material Gradient And Geometry Of Thick Hollow Cylinder

Consider an axisymmetric thick hollow cylinder as shown in Fig. 1, where a and b are the inner and outer radius of the cylinder, L is the length, r and z are the axis of cylindrical coordinate system.

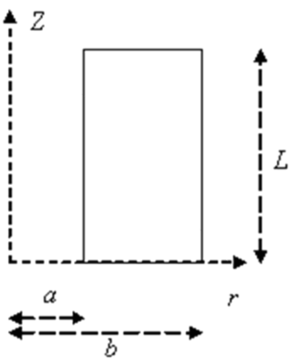

Fig. 1. Geometry of the cylinder

The cylinder's material is graded through the radial direction. The cylinder is made of a combined ceramic-metal material and the material composition varying continuously along its thickness (*r* direction) according to a power law distribution. The inner surface of the cylinder is pure ceramic and the outer surface is pure metal. The material distribution can be expressed as

$$P = P_c + (P_m - P_c)\left(\frac{r-a}{b-a}\right)^n \tag{1}$$

where P is temperature independent material property such as coefficient of heat conduction k, mass density ρ, thermal expansion coefficient α, specific heat capacity C, Young's modulus E. n is a non-negative volume fraction exponent, subscripts "*c*" and "*m*" stand for ceramic and metal.

2.2. Heat Conduction Equation

Consider a thick hollow cylinder as shown in Fig. 1. The formulation reduces to two dimensions, when the boundary conditions, loads and material properties are axisymmetric, so the formulation is independent of circumferential direction and cylindrical coordinates (*r,z*) are used. Heat conduction equation in axisymmetic cylindrical coordinates when the rate of heat generation is zero is as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_r(r)\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(k_z(r)\frac{\partial T}{\partial z}\right) = \rho(r)C(r)\,\partial T/\partial t \tag{2}$$

where $k_r(r)$, $k_z(r)$ are the coefficients of heat conduction in radial and axial directions and T denotes the temperature.

Thermal boundary conditions which are used in this research are

$$T(r, z, 0) = T_0 \tag{3-1}$$

$$T(r_{in}, z, t) = T_0 + T_0 \sin\left(\frac{\pi z}{L}\right)\left(1 - e^{(c_0 t)}\right) \qquad t \le 0.01\ (s) \tag{3-2}$$

$$T(r,0,t) = T(r,L,t) = T(r_{out},z,t) = T_0 \tag{3-3}$$

where T_0 is the reference temperature and c_0 is a constant value. r_{out} is the radius of points at the outer surface. The temperature boundary conditions denote that the inner surface of cylinder which is made ceramic is subjected to thermal shock and the upper, lower and outer surface of cylinder is kept in constant temperature.

2.3. Thermoelasticity Equations

Based on classical theory of linear thermoelasticity, the formulations for axisymmetric problem are expressed. The equations of motion in terms of stresses, disregarding the body forces are

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho(r)\frac{\partial^2 u}{\partial t^2} \tag{4-1}$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\tau_{rz}}{r} = \rho(r)\frac{\partial^2 v}{\partial t^2} \tag{4-2}$$

The stress and strain matrices are

$$[\sigma] = \begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \tau_{rz} \end{bmatrix} \tag{5}$$

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ \varepsilon_{rz} \end{bmatrix} \tag{6}$$

The strain-displacement relations for this analysis is

$$\varepsilon_{rr} = \frac{\partial u}{\partial r} \tag{7-1}$$

$$\varepsilon_{\theta\theta} = \frac{u}{r} \tag{7-2}$$

$$\varepsilon_{zz} = \frac{\partial v}{\partial z} \tag{7-3}$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right) \tag{7-4}$$

where u and v are the displacement components along the coordinates r , z. Above equations could be written in matrix form as

$$[\varepsilon] = [d][q] \tag{8-1}$$

$$[q] = \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{8-2}$$

$$[d] = \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ \frac{1}{r} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{1}{2}\frac{\partial}{\partial z} & \frac{1}{2}\frac{\partial}{\partial r} \end{bmatrix} \tag{8-3}$$

The stress- strain relations for axisymmetric condition are [36]

$$\sigma_{rr} = \frac{E(r)}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{rr} + \nu(\varepsilon_{\theta\theta} + \varepsilon_{zz})] - \frac{E(r)\alpha(r)\Delta T}{1-2\nu} \tag{9-1}$$

$$\sigma_{\theta\theta} = \frac{E(r)}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{\theta\theta} + \nu(\varepsilon_{rr} + \varepsilon_{zz})] - \frac{E(r)\alpha(r)\Delta T}{1-2\nu} \tag{9-2}$$

$$\sigma_{zz} = \frac{E(r)}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_{zz} + \nu(\varepsilon_{rr} + \varepsilon_{\theta\theta})] - \frac{E(r)\alpha(r)\Delta T}{1-2\nu} \tag{9-3}$$

$$\tau_{rz} = \frac{E(r)}{1+\nu}\varepsilon_{rz} \tag{9-4}$$

where $\Delta T = T - T_0$ is the temperature change distribution determined from the heat conduction equation. Stress– strain relations due to the temperature change in matrix form is as [36]

$$[\sigma] = [D^d]([\varepsilon] - [\varepsilon_T]) \tag{10}$$

$$[\varepsilon_T] = \alpha(r)\Delta T \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \tag{11}$$

where $[\varepsilon]$ denotes the elastic strain and $[\varepsilon_T]$ stands for the thermal strain due to the temperature change. The coefficients of elasticity $[D^d]$ is [36]

$$[D^d] = \frac{E(r)}{(1+\upsilon)(1-2\upsilon)} \begin{bmatrix} 1-\upsilon & \upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & \upsilon & 0 \\ \upsilon & \upsilon & 1-\upsilon & 0 \\ 0 & 0 & 0 & \frac{1-2\upsilon}{2} \end{bmatrix} \tag{12}$$

The cylinder is simply supported on its ends, so the displacement boundary conditions which are used in this research are assumed to be

$$u(r,0) = u(r,L) = 0 \tag{13}$$

3. FINITE ELEMENT MODELING

In order to solve governing equations, graded finite element method is used. Using graded elements causes continues variation of material properties than the homogenous elements. In this method, temperature, displacements and material properties interpolated using the same shape functions. In conventional finite element method, material properties in each element or layer were considered as constant values and the continuity conditions between each layer must be satisfied. So, the graded elements give a more continuous stress distribution than homogenous elements.

The finite element approximation of the domain is in the rz plane, which is the plane of revolution. To solve the heat conduction equation, the solution domain is divided into a number of simplex linear triangular elements. With a Kantrovich approximation for the time and space domain, the temperature distribution in the base element (e) is approximated as

$$T(r,z,t)^{(e)} = [N(r,z)^t]^{(e)}[T(t)]^{(e)} \tag{14-1}$$

$$[N(r,z)^t]^{(e)} = \begin{bmatrix} N_i \; N_j \; N_k \end{bmatrix} \tag{14-2}$$

$$[T(t)]^{(e)} = \begin{bmatrix} T(t)_i \\ T(t)_j \\ T(t)_k \end{bmatrix} \tag{14-3}$$

where $[N(r,z)^t]^{(e)}$ is the matrix of linear interpolation functions in terms of its nodal values in element (e) and $[T(t)]$ is the nodal temperature vector of element. Indice *t* stands for the temperature solution.

It can be simply verified that the extremum of “(2)” reduces to “(15)”. So, the associated functional of “(2)” is as [36]

$$I = \int_V \frac{1}{2}\left[rk_r\left(\frac{\partial T}{\partial r}\right)^2 + rk_z\left(\frac{\partial T}{\partial z}\right)^2 - 2\left(Q - \rho C\frac{\partial T}{\partial t}\right)T\right]dV + \int_{S_1} q^{"}TdS + \int_{S_2} \frac{1}{2}h(T - T_\infty)^2 dS \tag{15}$$

where V is the volume of the cylinder, S_1 and S_2 are the boundary surfaces of the cylinder which are subjected to the heat flux and heat convection, $q^{"}$ and Q are the heat flux and the rate of energy generation per unit volume per unit time, T_∞ and h are the ambient temperature and heat convection coefficient.

Finite element approximation is followed by applying the Ritz method to the "(1)". The details of this method could be found in [36]. Using variational formulation, capacitance matrix $[{K_1}^t]^{(e)}$, stiffness matrices $[{K_2}^t]^{(e)}$ and $[{K_3}^t]^{(e)}$ and force matrix $[F^t]$ for each element (*e*) are obtained as

$$[{K_1}^t]^{(e)} = \int_{V(e)} \rho(r)C(r)[N^t]^T[N^t]dV \tag{16}$$

$$[{K_2}^t]^{(e)} = \int_{V(e)} [B^t]^T[D(r)^t][B^t]\, dV \tag{17-1)}$$

where

$$[D(r)^t] = \begin{bmatrix} rk_r(x) & 0 \\ 0 & rk_z(x) \end{bmatrix} \tag{17-2)}$$

$$[B^t] = \begin{bmatrix} \frac{\partial N_i}{\partial r} & \frac{\partial N_j}{\partial r} & \frac{\partial N_k}{\partial r} \\ \frac{\partial N_i}{\partial z} & \frac{\partial N_j}{\partial z} & \frac{\partial N_k}{\partial z} \end{bmatrix} \tag{17-3)}$$

$$[{K_3}^t]^{(e)} = \int_{S(e)} h\,[N^t]^T[N^t]dS \tag{18}$$

$$[F^t]^{(e)} = \int_{S_2(e)} hT_\infty[N^t]^T dS + \int_{V(e)} Q[N^t]^T dV - \int_{S_1(e)} q^{"}[N^t]^T dS \tag{19}$$

where *V(e)*, $S_1(e)$ and $S_2(e)$ are the volume of element, boundary of the element which is subjected to heat flux and heat convection, respectively. In this research, force matrices associated with the temperature solution are assumed to be zero and the temperature distribution due to heat conduction is considered.

Now by assembling the element matrices, the global matrix equation of heat transfer for the functionally graded cylinder can be obtained as

$$[{K_1}^t][\dot{\theta}] + \left([{K_2}^t] + [{K_3}^t]\right)[\theta] = [F^t] \tag{20}$$

Direct integration method may be used to integrate "(20)" in time domain. The Crank- Nicolson method [36] with suitable time step is used which is unconditionally stable algorithm.

As the transient temperature distribution derived, the thermoelasticity problem can be solved. To approximate the displacement components, the simplex linear triangular elements are also used. Assuming the shape function $[N(r,z)^d]^{(e)}$ for the displacement vector, the displacement matrix for each element (e) in terms of the nodal displacement matrix $[\delta]$ and shape function $[N(r,z)^d]^{(e)}$ is

$$[q]^{(e)} = [N(r,z)^d]^{(e)}[\delta]^{(e)} \tag{21-1}$$

$$[\delta]^{(e)} = \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{bmatrix} \tag{21-2}$$

The indice *d* stands for thermoelasticity solution of the problem. Substituting “(21-2)” in “(8-1)” gives the elastic strain matrix of element (e) as

$$[\varepsilon]^{(e)} = [B^d]^{(e)}[\delta]^{(e)} \tag{22}$$

where $[B^d]^{(e)} = [d][N(r,z)^d]^{(e)}$ and is essential for computation of the structural stiffness and the thermal stress matrices at each element. Due to the nature of operator matrix $[d]$ in cylindrical coordinates, matrix $[B^d]$ is in general a function of the variable *r*. For the simplex linear triangular element, matrices$[N(r,z)^d]^{(e)}$ and $[B]^{(e)}$ are

$$[N(r,z)^d]^{(e)} = \begin{bmatrix} N_i & 0 & N_j & 0 & N_k & 0 \\ 0 & N_i & 0 & N_j & 0 & N_k \end{bmatrix} \tag{23}$$

$$[B^d]^{(e)} = \frac{1}{2A}\begin{bmatrix} b_i & 0 & b_j & 0 & b_k & 0 \\ \frac{2AN_i}{r} & 0 & \frac{2AN_j}{r} & 0 & \frac{2AN_k}{r} & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ \frac{c_i}{2} & \frac{b_i}{2} & \frac{c_j}{2} & \frac{b_j}{2} & \frac{c_k}{2} & \frac{b_k}{2} \end{bmatrix} \tag{24}$$

The components of matrix $[N(r,z)^d]^{(e)}$ are given in [36].

The finite element model of the governing equations of thermoelasticity problem can be derived using Rayleigh Ritz energy formulation. The details of this method could be found in [36]. By applying this method to the governing equations, the structural stiffness, force and mass element matrices due to temperature change can be written as

$$[K^d]^{(e)} = \int_{V_{(e)}} [B^d]^T[D^d][B^d]dV \tag{25}$$

$$[F^d]^{(e)} = \int_{V_{(e)}} [B^d]\,[D^d][\varepsilon_T]\,dV \tag{26}$$

$$[M^d]^{(e)} = \int_{V_{(e)}} \rho[N]^T[N]dV \tag{27}$$

Now by assembling the element matrices, the global motion equations for the FG thick cylinder can be obtained as

$$[M^d]\{\ddot{\Delta}\}[K^d][\Delta] = \{F^d\} \tag{28}$$

Using Newmark direct integration method [36] and the known temperature change which is derived from "(20)," "(28)" is solved. As the variational formulation is applied, the stress boundary conditions in "(28)" are implicitly satisfied.

$$\sigma_{rr}(r_{in}\,, z) = \sigma_{rr}(r_{out}\,, z) = \tau_{rz}(r_{in}\,, z) = \tau_{rz}(r_{out}\,, z) = \sigma_{zz}(r, 0) = \sigma_{zz}(r, L) = 0 \tag{29}$$

It should be noted as the graded elements are used to model the problem, the material properties interpolated using the shape functions like what used for the temperature and displacements.

$$P = \sum_{i=1}^{3} P_i N_i \tag{30}$$

where P_i is the material property corresponding to node i.

4. Results and discussions

4.1. Validation

To show the validity of the presented method, we performed the finite element analysis for a finite length simply-supported functionally graded hollow cylinder which is subjected to an axisymmetric steady state temperature loading. Shao [23] using a multi layered approach based on the theory of laminated composites analytically solved this problem. Consider the FG thick hollow cylinder of ref [23] with inner radius $a=0.7$ m, outer radius $b=1$ m and length $L=5$ m. Coefficient of heat conduction and thermal expansion coefficient at the inner and outer surface are $k_c = 5.9W.(mK)^{-1}$, $\alpha_c = 4.8*10^{-6} K^{-1}$, $k_m = 138W.(mK)^{-1}$ and $\alpha_m = 4.9*10^{-6} K^{-1}$. The steady state thermal boundary conditions were assumed to be

$$T\left(a,z\right) = 200\sin(\frac{\pi z}{L})$$

$$T\left(b,z\right) = T\left(r,0\right) = T\left(r,L\right) = 0$$

This problem could also be solved by the graded finite element method developed here as a transient problem. So, the transient temperature loading at the inner surface of the cylinder and initial temperature condition are considered as

$$T\left(a,z,t\right) = 200\sin(\frac{\pi z}{L})\left(1-e^{(c_0 t)}\right)$$

$$T\left(r,z,0\right) = 0$$

where c_0 is a constant value and is considered as $c_0 = -2$.

The distribution of temperature through the cylinder for long time and the power law exponent *n=5* is shown in Figure 2. The temperature distribution at $\frac{z}{L} = 0.5$ compared with result of [23] in Fig. 2. A good agreement between these two results is observed.

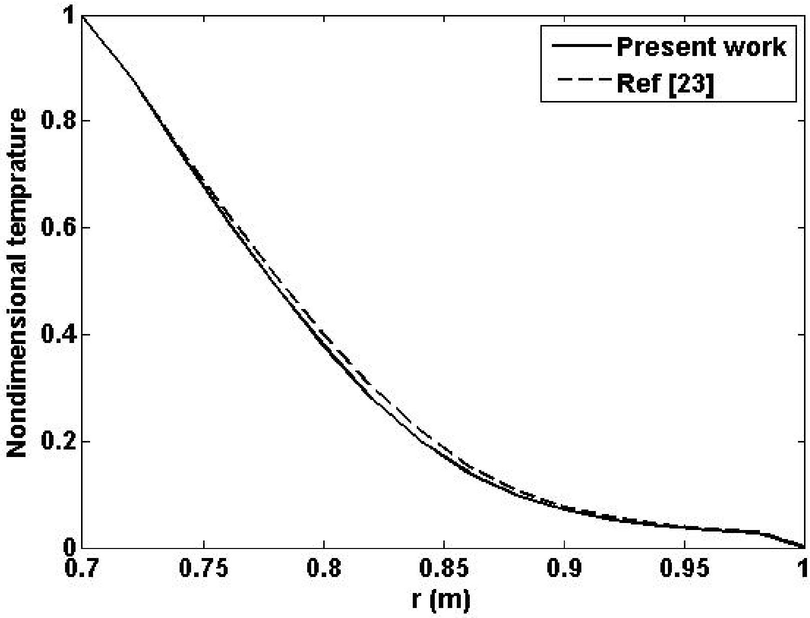

Fig. 2. Temperature distribution at $\frac{z}{L} = 0.5$ and *n=5* compared with [23]

4.2. Numerical Results And Discussion

A thick hollow cylinder with inner radius *a=1 m* and outer radius *b=1.5 m* and length *L=1 m* is considered here. The cylinder's material is graded through the thickness direction according to the "(1-1)". The inner surface of the cylinder is made of pure ceramic (Alumina) and the thermoelastic properties at the inner surface are $k_c = 46\ (\frac{w}{m.k})$, $\alpha_c = 7.4 * 10^{-6}\ (\frac{1}{k})$, $C_c = 760\ \left(\frac{j}{Kg.k}\right)$, $\rho_c = 3800\ (\frac{Kg}{m^3})$ and $E_c = 380\ (Gpa)$. Also the outer surface of the cylinder is made of pure metal (Aluminum) and its material properties are considered as: $k_m = 250\ (\frac{w}{m.k})$, $\alpha_m = 23 * 10^{-6}\ (\frac{1}{k})$, $C_m = 896\ \left(\frac{j}{Kg.k}\right)$, $\rho_m = 2707\ (\frac{Kg}{m^3})$ and $E_m = 70\ (Gpa)$. The Poisson's ratio is assumed to be constant and is considered as $\nu = 0.3$. The cylinder is simply supported and thermally loaded according to the "(3-1)-(3-3)". So the reference temperature $T_0 = 300\ (k)$ and the constant value $c_0 = -100$. In this section, responses using graded finite element method are obtained and the distribution of displacements and stresses for different distribution of material properties are presented.

Figs. 3 and 4 show the distribution of radial displacement of cylinder at t=0.0021 s for power law exponent *n=1*and 5, respectively. Also, Figs. 5 and 6 show the distribution of radial displacement of cylinder at t=0.0084 s for power law exponent *n=1*and 5, respectively. As it can be from these results the displacement boundary conditions are satisfied correctly. Figs. 7 and 8 show the distribution of axial displacement of cylinder at t=0.0021 s for power law exponent *n=1*and 5, respectively. Also, Figs. 9 and 10 show the distribution of axial displacement of cylinder at t=0.0084 s for power law exponent *n=1*and 5, respectively. These results denote that due to applying thermal shock to the cylinder, the cylinder shows a vibrational behavior called thermal induced vibration, and displacement component shows a dynamic behavior which is excited by thermal shock at inner surface of cylinder.

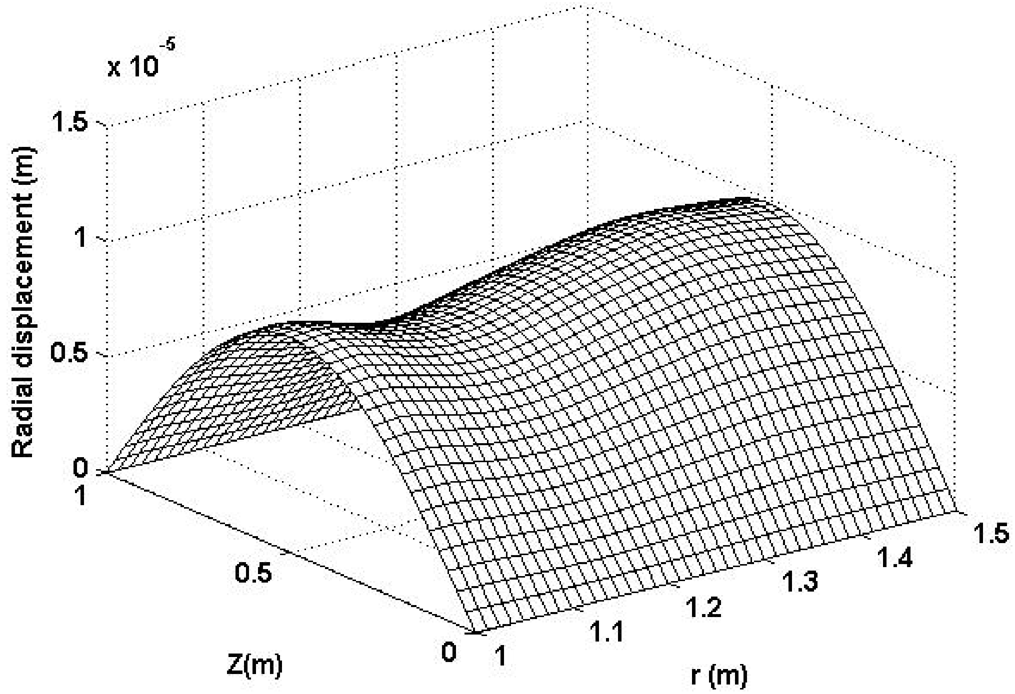

Fig. 3. **Radial displacement distribution through the cylinder for *n=1* at *t=0.0021 (s)***

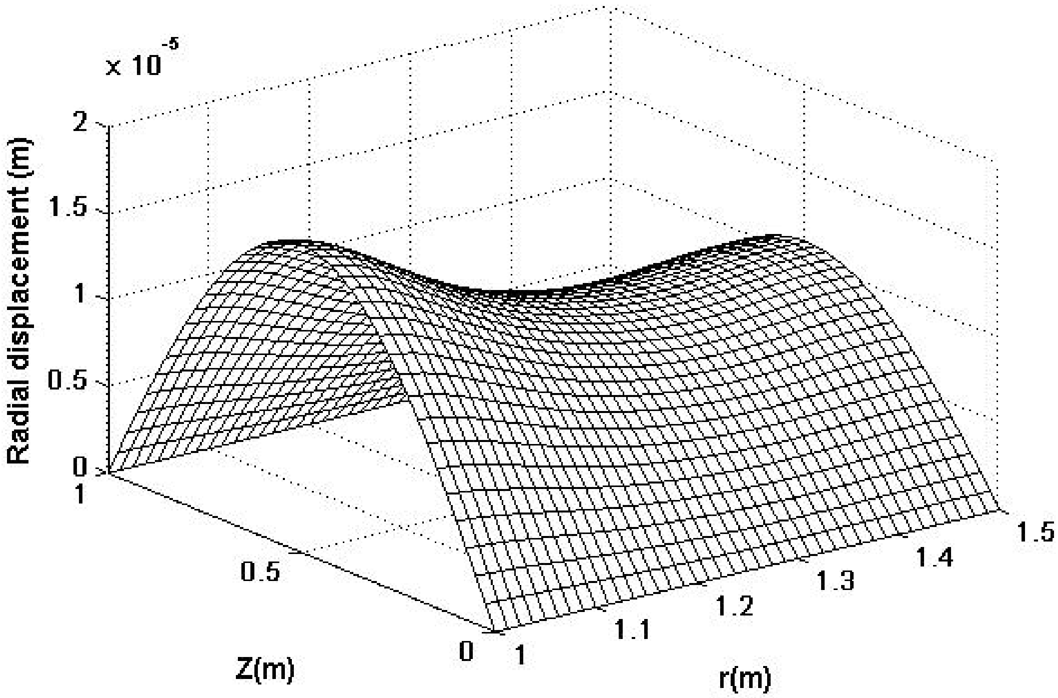

Fig. 4. **Radial displacement distribution through the cylinder for *n=5* at *t=0.0021 (s)***

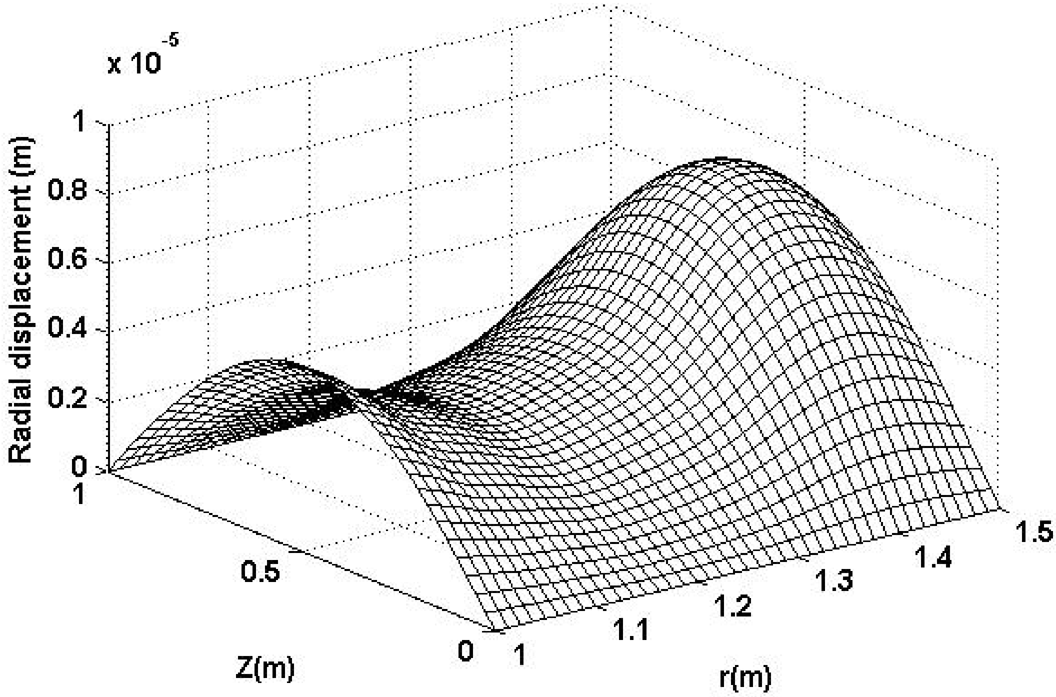

Fig. 5. **Radial displacement distribution through the cylinder for *n=1* at *t=0.0084 (s)***

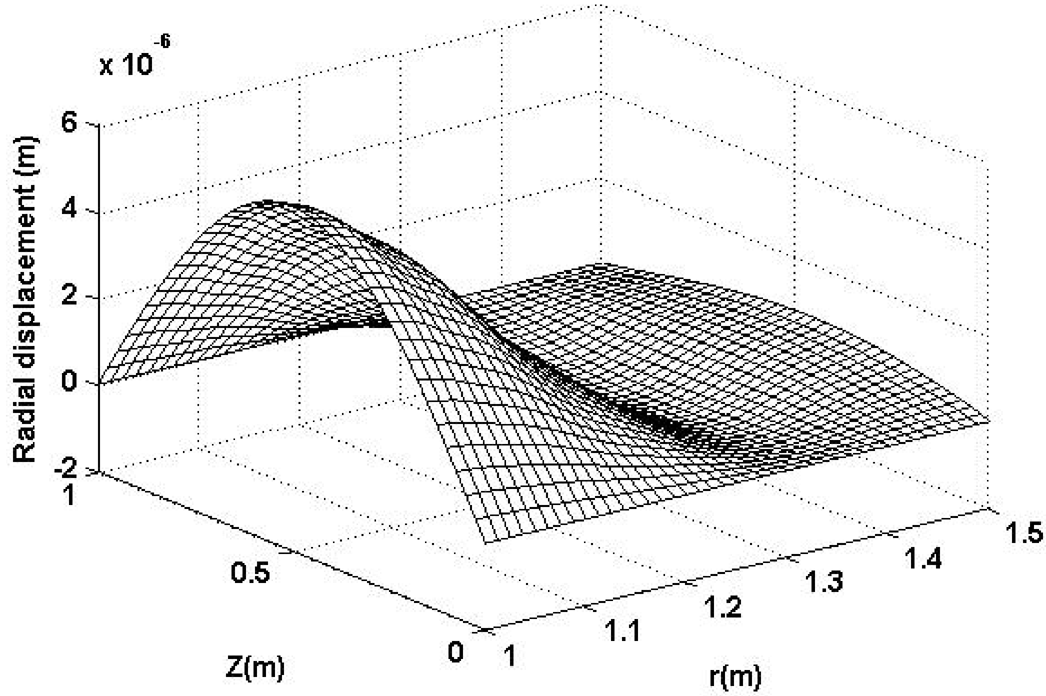

Fig. 5. **Radial displacement distribution through the cylinder for *n=5* at *t=0.0084 (s)***

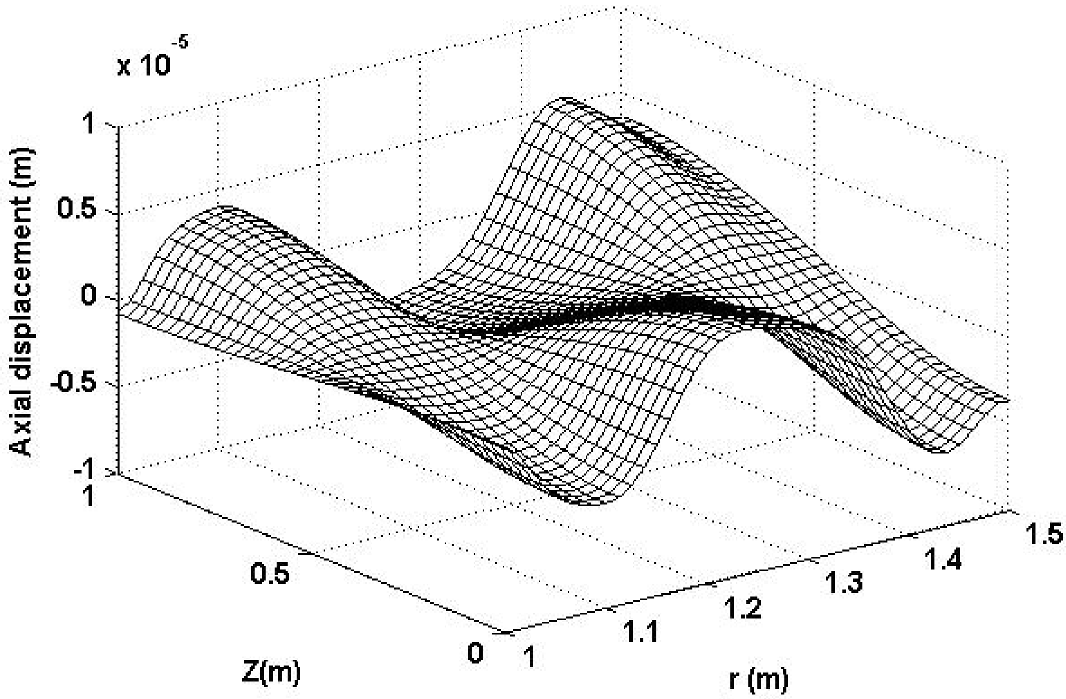

Fig. 7. Axial displacement distribution through the cylinder for *n=1* at *t=0.0021 (s)*

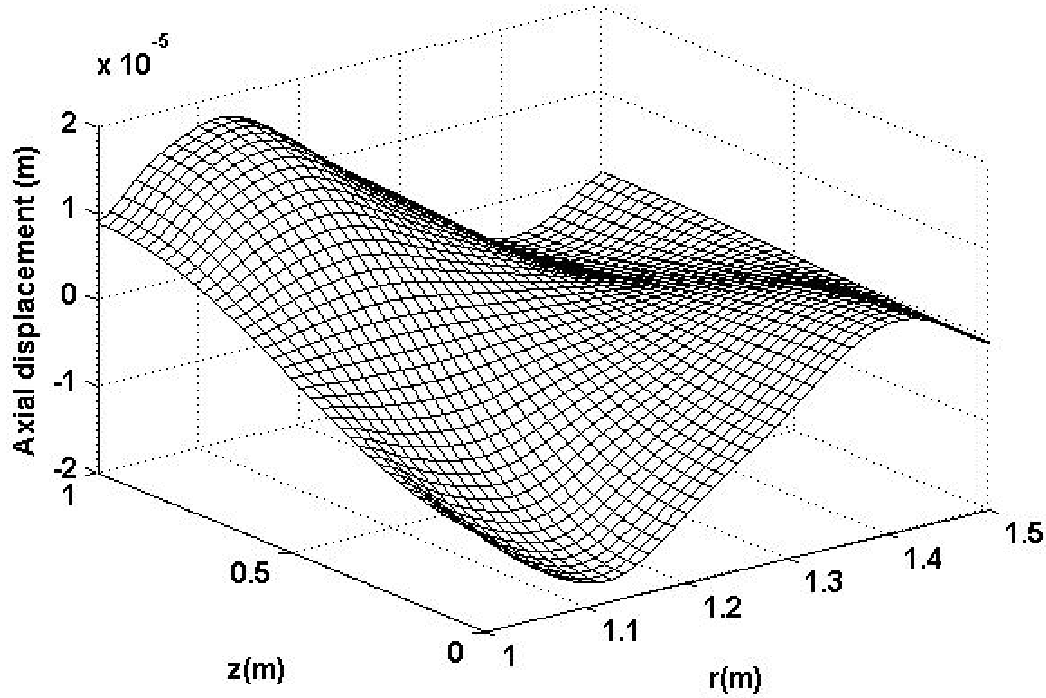

Fig. 8. Axial displacement distribution through the cylinder for *n=5* at *t=0.0021 (s)*

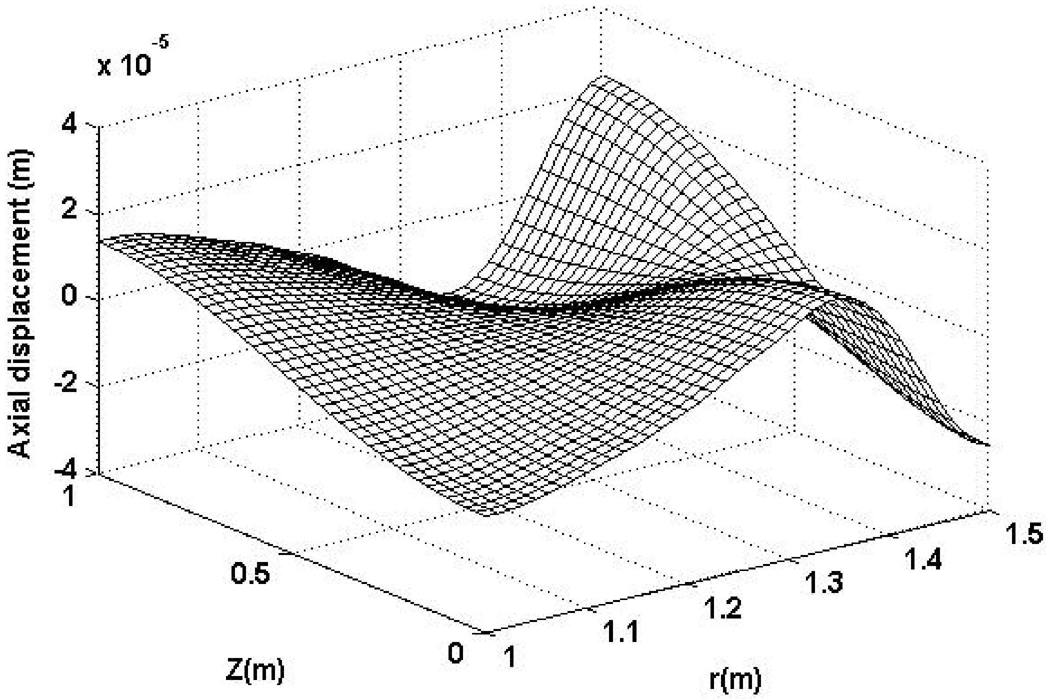

Fig. 9. **Axial displacement distribution through the cylinder for *n=1* at *t=0.0084 (s)***

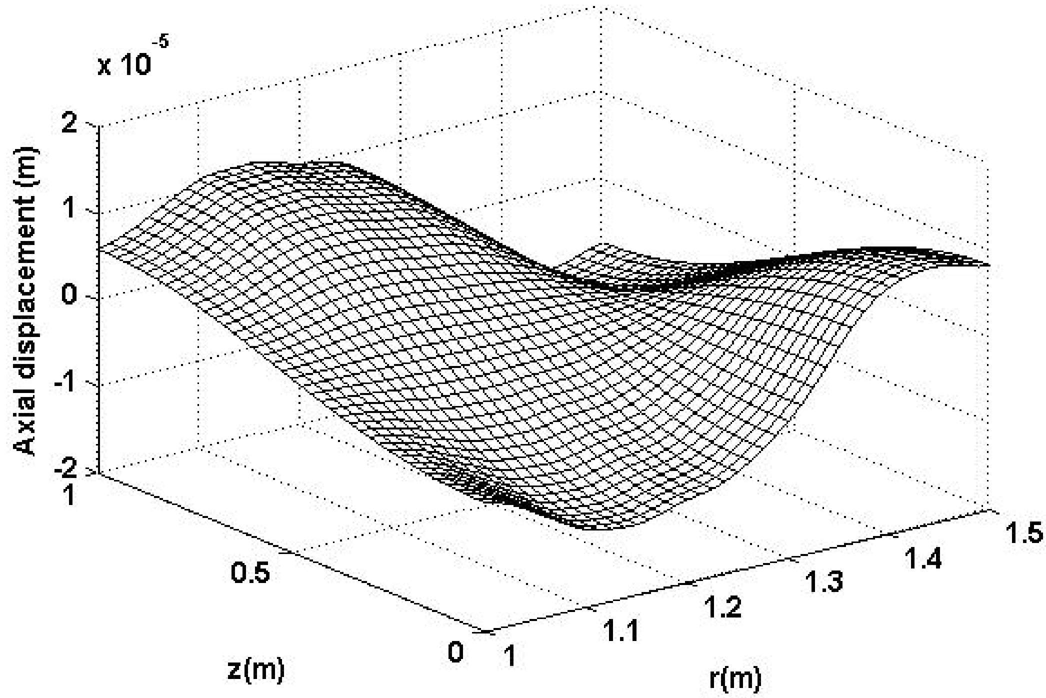

Fig. 10. **Axial displacement distribution through the cylinder for *n=5* at *t=0.0084 (s)***

Figs. 11 and 12 show the distribution of radial stress of cylinder at t=0.0084 s for power law exponent *n=1*and 5, respectively. The same results for axial stress are presented in Figs 13 and 14. Also, Figs. 15 and 16 show distribution of shear stress of cylinder at t=0.0019 s for power law exponent *n=1*and 5, respectively.

These results show that the natural boundary conditions in "(29)" are well satisfied. As it can be seen from these results the distribution of stresses has continuous and smooth variation due to using graded elements.

Overview of obtained results in this paper denotes that the distribution of displacements and thermal stresses through the functionally graded cylinder are significantly affected by the material distribution. These results can be considered in design optimization problems and help engineers to design the thick hollow cylinder made of FGMs with desired stress distributions.

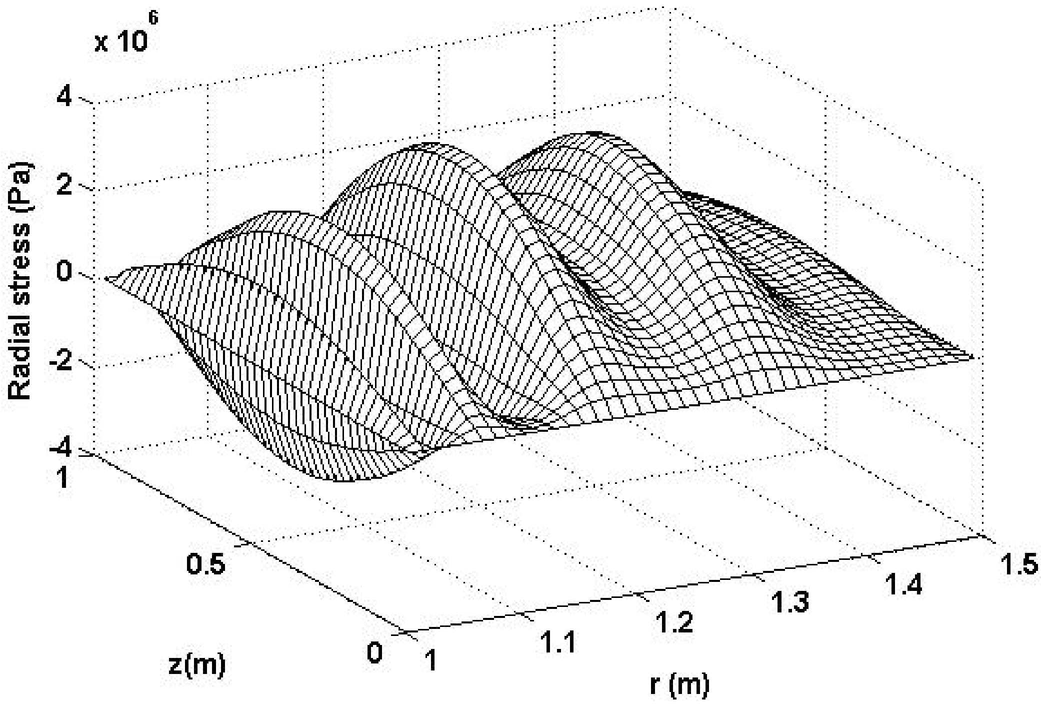

Fig. 11. Distribution of radial stress through the cylinder for and *n=1* at *t=0.0084 (s)*

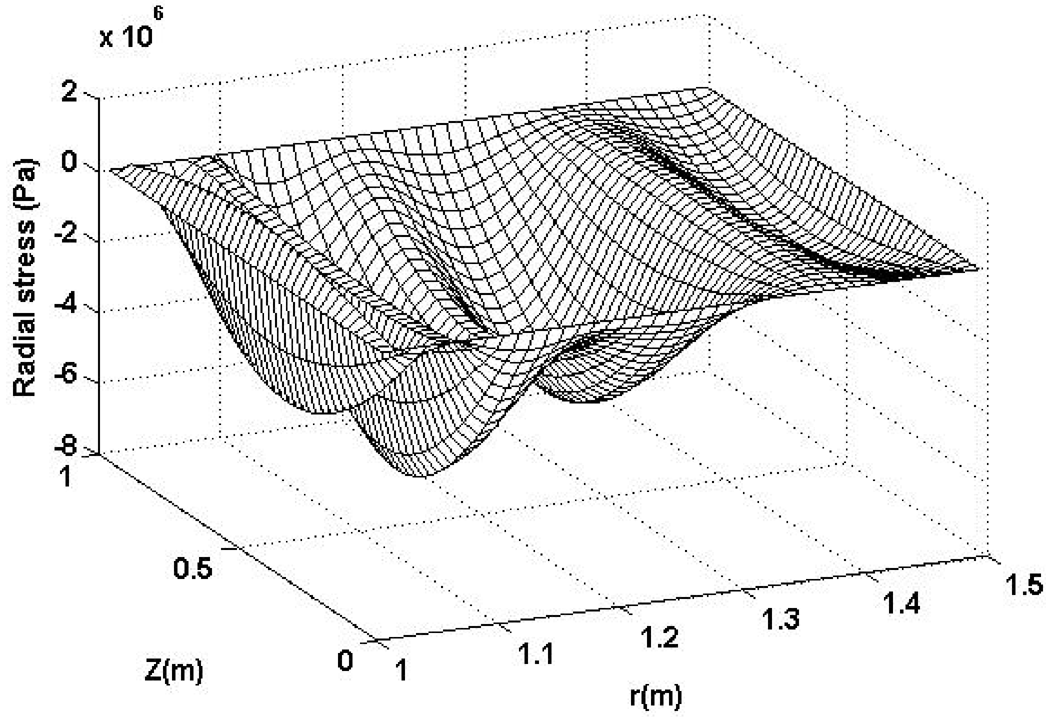

Fig. 12. Distribution of radial stress through the cylinder for and *n=5* at *t=0.0084 (s)*

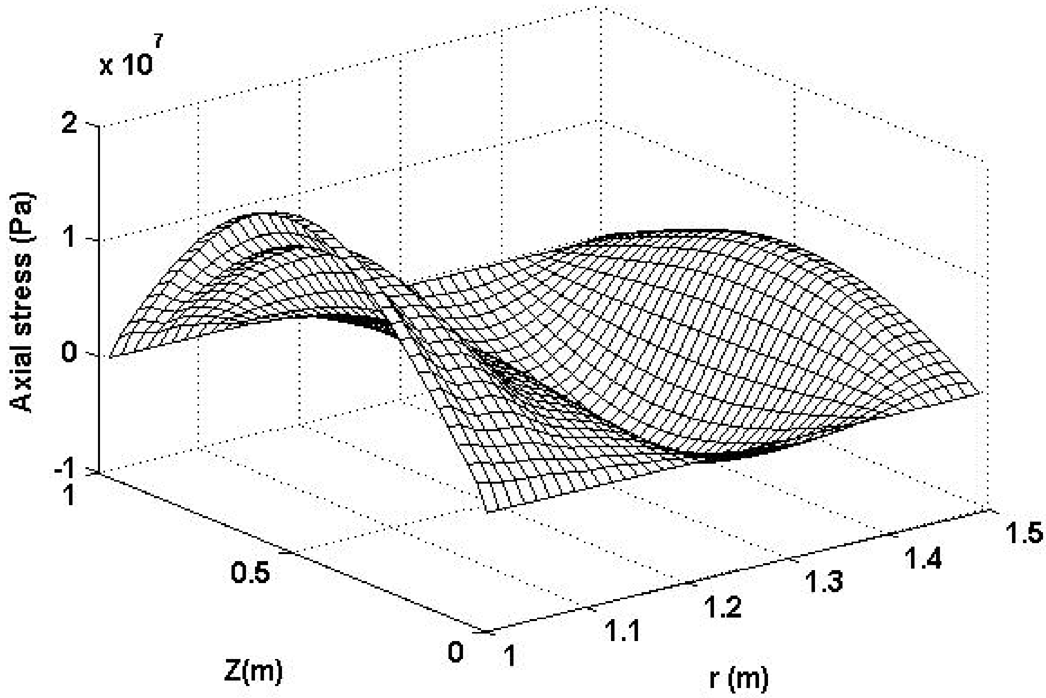

Fig. 13. Distribution of axial stress through the cylinder for and *n=1* at *t=0.0084 (s)*

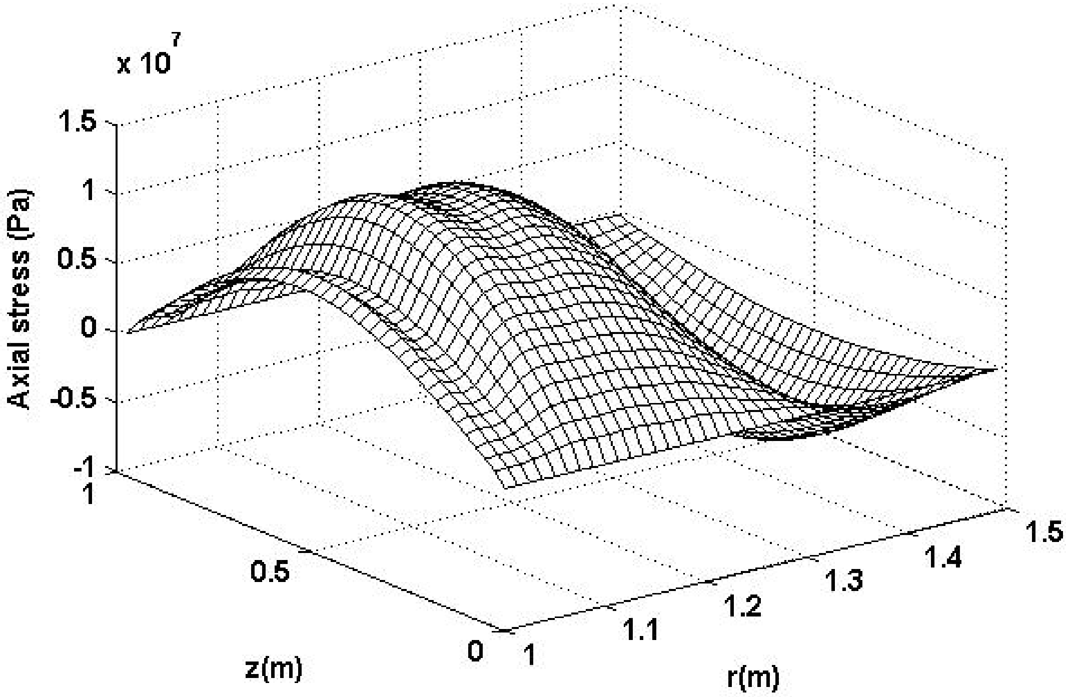

Fig. 14. Distribution of axial stress through the cylinder for and *n=5* at *t=0.0084 (s)*

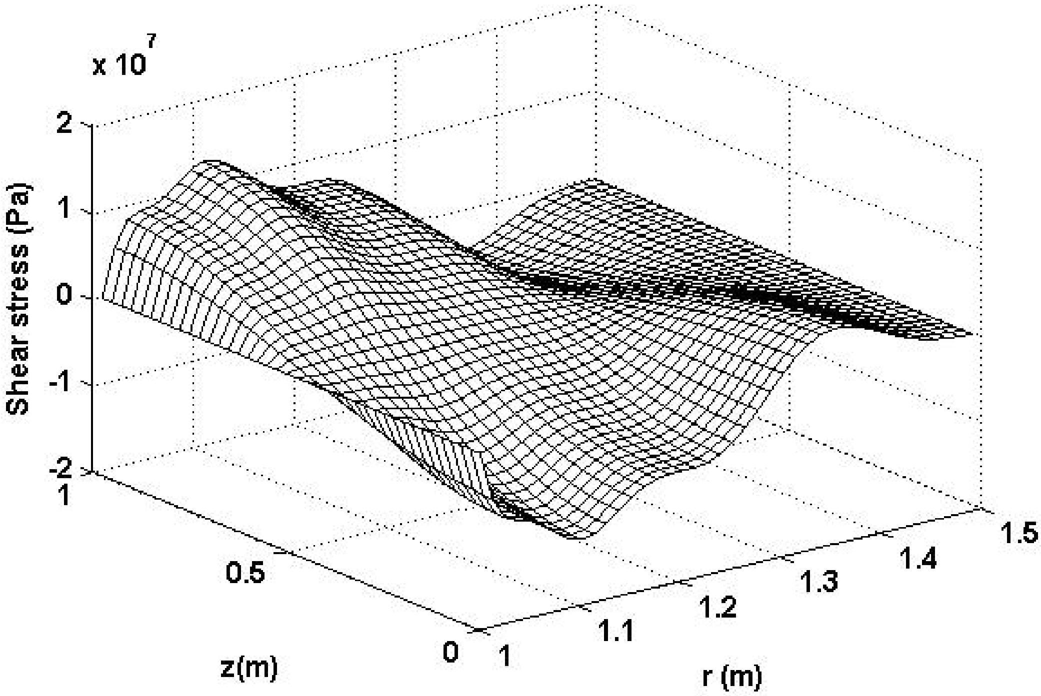

Fig. 15. Distribution of shear stress through the cylinder for *n=1* at *t=0.0019 (s)*

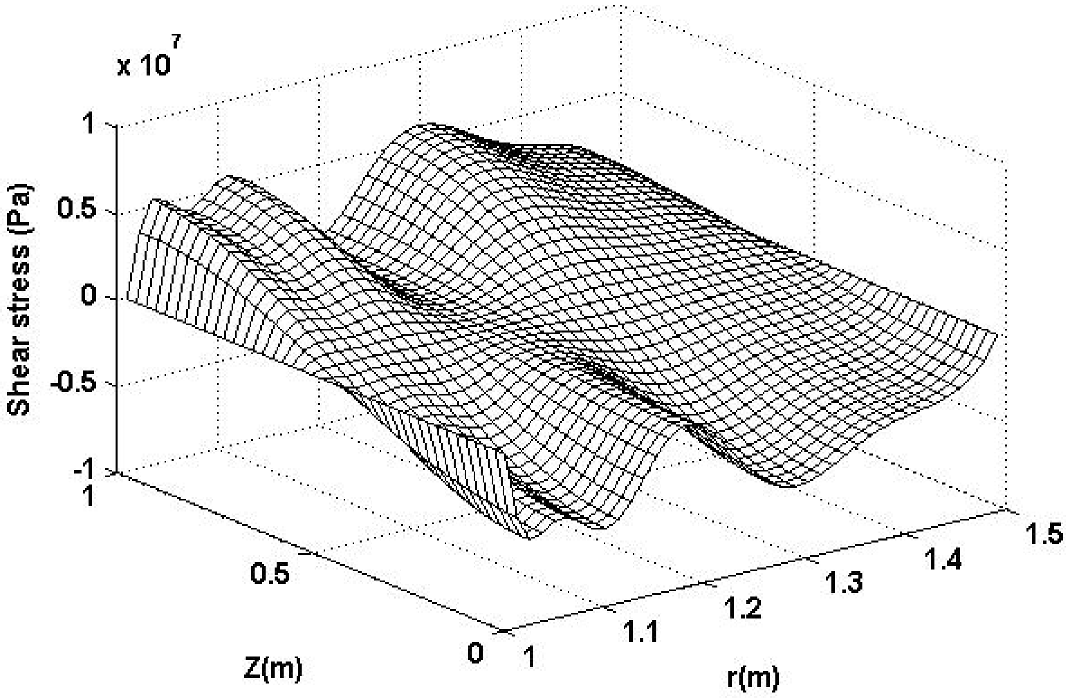

Fig. 16. Distribution of shear stress through the cylinder for *n=5* at *t=0.0019 (s)*

5. CONCLUSIONS

This paper presents dynamic thermal stresses based on the classical theory of linear thermo-elasticity in FGM thick hollow cylinders. Graded finite element method based on Rayleigh-Ritz energy formulation is applied to model the problem. Results denote that using graded elements are more advantageous than homogenous elements to model the FGM structures and gives continuous stress field. The present solution can be used for all types of axisymmetric structures like thick hollow cylindrical and truncated conical shells with finite and infinite lengths, and also various loading and boundary conditions and different functions which are suggested for modeling FGMs can be applied to these problems.

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